

# Study of $B_s \rightarrow \phi \ell^+ \ell^-$ Decay in a Single Universal Extra Dimension

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## Abstract

Utilizing form factors calculated within the light-cone sum rules, we have evaluated the decay branching ratios of  $B_s \rightarrow \phi \gamma$  and  $B_s \rightarrow \phi \ell^+ \ell^-$  in a single universal extra dimension model (UED), which is viewed as one of the alternative theories beyond the standard model (SM). For the decay  $B_s \rightarrow \phi \ell^+ \ell^-$ , the dilepton invariant mass spectra, the forward-backward asymmetry, and double lepton polarization are also calculated. For each case, we compared the obtained results with predictions of the SM. In lower values of the compactification factor  $1/R$ , the only parameter in this model, we see the considerable discrepancy between the UED and SM models. However, when  $1/R$  increases, the results of UED tend to diminish and at  $1/R = 1000$  GeV, two models have approximately the same predictions. Compared with data from CDF of  $B_s \rightarrow \phi \mu^+ \mu^-$ , the  $1/R$  tends to be larger than 350 GeV. We also note that the zero crossing point of the forward-backward asymmetry is become smaller, which will be an important plat to prob the contribution from the extra dimension model. The results obtained in this work will be very useful in searching new physics beyond SM. Moreover, the order of magnitude for branching ratios shows a possibility to study these channels at the Large Hadron Collider (LHC), CDF and the future super-B factory.

## 1 Introduction

As one kind of the flavor changing neutral current processes, the electroweak penguin decays  $b \rightarrow s \ell^+ \ell^-$  appearing only at the loop level in the standard model (SM), are therefore sensitive to the fine structure of SM and to the possible new physics as well, and are expected to shed light on the existence of new physics before the possible new particles are produced at colliders. With the data from Belle and BaBar experiments, the exclusive processes  $B \rightarrow K^{(*)} \ell^+ \ell^-$  have received great interest so that their theoretical calculation has been the subject of many investigations in the SM [1, 2] and beyond [3, 4]. Along this line, the exclusive decays  $B_s \rightarrow \phi \ell^+ \ell^-$  become also attractive since these decays are also induced by  $b \rightarrow s \ell^+ \ell^-$ , and could be measured at the running Tavatron, LHC and future super-B factories. Recently, the CDF collaboration had observed the rare semi-leptonic decay  $B_s \rightarrow \phi \mu^+ \mu^-$  [5] and the branching ratio is

$$\text{Br}(B_s \rightarrow \phi \mu^+ \mu^-) = [1.44 \pm 0.33(\text{stat.}) \pm 0.46(\text{syst.})] \times 10^{-6}. \quad (1)$$

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This exclusive process is quite worthy of intensive research and have attached much attention [6, 7]. When studying the semi-leptonic decays, dilepton invariant mass spectrum, the forward-backward asymmetry, and double lepton polarization are important observables to test SM and prob new physics, while the first two are mostly analyzed. Due to small mass of electron and muon, the invariant mass spectra and branching ratios are almost the same for electron and muon modes. Meanwhile, it is very difficult to measure the electron polarization, so we only consider  $B_s \rightarrow \phi \mu^+ \mu^-, \tau^+ \tau^-$  in this work.

To make the theoretical predictions clearly, additional knowledge of decay form factors is needed, which is related with the calculation of hadronic transition matrix elements, and can only be reliably calculated by using a non-perturbative QCD method. Fortunately, these form factors related to  $B_s \rightarrow \phi \ell^+ \ell^-$  have been explored within the different methods, such as light cone sum rules (LCSRs) [8, 9, 10], perturbative QCD approach [11] and in different quark models: relativistic constitute quark model [12], constituent quark model [13], light front quark model [14]. Among them, the light-cone sum rules, which deal with form factors at small momentum region, is complementary to the lattice approach and has consistence with perturbative QCD and the heavy quark limit. We will adopt the form factors calculated by the LCSR.

Among the ideas proposed to extend the SM, a lot of attention has recently been devoted to models including extra dimensions [15, 16]. An interesting model is that proposed by Appelquist, Cheng and Dobrescu with the so-called universal extra dimensions (UED) [17], which means that all the SM fields may propagate in one or more compact extra dimensions. The compactification of the extra dimensions involves the appearance of an infinite discrete set of four dimensional fields which create the so-called KK particles. The simplest UED scenario is characterized by a single extra dimension. Compared to the SM, this model has one extra parameter called compactification radius,  $R$ . Hence, this model is a minimal extension of the SM in  $4 + 1$  dimensions with the extra dimension compactified to the orbifold  $S^1/Z_2$  and the fifth coordinate,  $y$  running between 0 and  $2\pi R$ , and  $y = 0$  and  $y = \pi R$  are fixed points of the orbifold. The zero modes of fields propagating in the extra dimension correspond the SM particles. The masses of KK particles are related to compactification radius according to the relation  $m_n^2 = m_0^2 + n^2/R^2$ , with  $n = 1, 2, \dots$ . One of the important property of the model is the conservation of KK parity that guarantees the absence of tree level KK contributions to low energy processes occurring at scales much smaller than the compactification scale.

After the UED model being proposed, many attempts have been done to constraint the only parameter compactification radius  $R$ , for example, from Tevatron experiments the bound on the inverse compactification radius is found to be about  $1/R \geq 300 \text{ GeV}$ . The anomalous magnetic moment of muon and  $Z \rightarrow \bar{b}b$  vertex also lead to the same conclusion. Rare  $B$  transitions can also be used to constrain this scenario. Buras and collaborators [18] have investigated the impact of universal extra dimensions on the  $B_{d,s}^0 - \bar{B}_{d,s}^0$  mixing mass differences, on the CKM unitarity triangle and on inclusive  $b \rightarrow s$  decays for which they have computed the effective Hamiltonian. In particular, it was found that  $BR(B \rightarrow X_s \gamma)$  allowed to constrain  $1/R > 250 \text{ GeV}$ , a bound updated by a more recent analysis to  $1/R > 600 \text{ GeV}$  at 95% CL, or to  $1/R > 330 \text{ GeV}$  at 99% CL [19]. In this work, we will consider the  $1/R$  from 200 GeV up to 1000 GeV. In the past years, the UED model has been applied widely to calculate many observables related to the radiative and semileptonic decays of hadrons (see for example [4, 7, 20, 21, 22, 23]).

The aim of the paper is to find the effects of the KK modes on various observables related to the  $B_s \rightarrow \phi \ell^+ \ell^-$  transition, and these observables involve the dilepton invariant mass spectra, the forward-backward asymmetry, and double lepton polarization. We will compare the obtained results with the predictions of the standard model. In Ref. [7], R. Mohanta and A. K. Giri had calculated the processes  $B_s \rightarrow \phi \ell^+ \ell^-$  and the  $B_s \rightarrow \gamma \ell^+ \ell^-$  under the UED model, where they calculated the branching ratios and forward-backward asymmetries adding the long distance contribution. In this work, we will drop the contribution from the resonances, such as  $J/\psi, \psi'$ , and recalculate all observables. Moreover, we will also calculate the lepton polarizations, which are always viewed as good places to prob the new physics contribution. In other words, this work can be regarded as supplementary of Ref.[7].

The outline of the paper is as follows. In section 2, after introducing the effective Hamiltonian responsible for the  $b \rightarrow s \ell^+ \ell^-$  transition and form factors of  $B_s \rightarrow \phi$ , we will present the formula of observable. In section 3, we numerically analyze the considered observables of  $B_s \rightarrow \phi \mu^+ \mu^-$ ,  $B_s \rightarrow \phi \tau^+ \tau^-$ . This section also includes a comparison of the results obtained in UED model with that predicted by the SM. We will summarize this work at last.

## 2 Effective Hamiltonian, Form Factors and Formula of Observable

At quark level, the  $B_s \rightarrow \phi \ell^+ \ell^-$  transition proceed via FCNC transition of the  $b \rightarrow s \ell^+ \ell^-$ . Neglecting the doubly Cabibbo-suppressed contributions, the effective Hamiltonian governing  $b \rightarrow s \ell^+ \ell^-$  transition is given by [24, 25]

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu), \quad (2)$$

where explicit expressions of  $O_i$  could be found in Ref. [24], and the Wilson coefficients  $C_i$  can be calculated perturbatively [26, 27, 28, 29]. Using the effective Hamiltonian, the free quark decay amplitude can be written as:

$$\begin{aligned} \mathcal{M} = & \frac{G_F \alpha_{em} V_{tb} V_{ts}^*}{2\sqrt{2}\pi} \left[ C_9^{eff} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu \ell + C_{10} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu \gamma_5 \ell \right. \\ & \left. - 2m_b C_7^{eff} \frac{q^\nu}{q^2} \bar{s} i \sigma_{\mu\nu} (1 + \gamma_5) b \bar{\ell} \gamma^\mu \ell \right]. \end{aligned} \quad (3)$$

Here  $q = p_+ + p_-$ , where  $p_\pm$  are the four momenta of the leptons, respectively. Noted that  $\mathcal{M}$ , although a free quark decay amplitude, contains part of long-distance effects from four-quark interactions, which usually are absorbed into a redefinition of the short distance Wilson coefficients. To be specific, we define the effective coefficient of the operator  $O_9$  as

$$C_9^{eff} = C_9 + Y(q^2), \quad (4)$$

where  $Y(q^2)$  stands for the above mentioned contribution from four-quark interaction. The corresponding operators and  $Y(q^2)$  can refer to Ref. [30]. In this work, as mentioned before, we have not consider the contribution mainly due to  $J/\psi$  and  $\psi'$  resonances in the decay chain  $B_s \rightarrow \phi \psi^{(\prime)} \rightarrow \phi \ell^+ \ell^-$ , which could be vetoed experimentally.

The main source of the deviation of the UED model and SM predictions on the considered observables are from Wilson coefficients  $C_7^{eff}$ ,  $C_9^{eff}$  and  $C_{10}$ , which can be expressed in terms of the periodic functions,  $F(x_t, 1/R)$  with

$x_t = m_t^2/M_W^2$  and  $m_t$  being the top quark mass. Similar to the mass of the KK particles described in terms of the zero modes ( $n = 0$ ) correspond to the ordinary particles of the SM and additional parts coming from the UED model, the functions,  $F(x_t, 1/R)$  are also written in terms of the corresponding SM functions,  $F_0(x_t)$  and additional parts which are functions of the compactification factor,  $1/R$ , i.e.,

$$F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n), \quad (5)$$

where  $x_n = \frac{m_n^2}{M_W^2}$  and  $m_n = \frac{n}{R}$ . The Glashow-Illiopoulos-Maiani (GIM) mechanism guarantees the finiteness of the functions,  $F(x_t, 1/R)$  and satisfies the condition,  $F(x_t, 1/R) \rightarrow F_0(x_t)$ , when  $R \rightarrow 0$ . As far as  $1/R$  is taken in the order of a few hundreds of  $GeV$ , the Wilson coefficients differ considerably from the SM values. For explicit expressions of the Wilson coefficients in UED model see [4, 18]. In Fig. 1, we plot the Wilson coefficients  $C_i$  ( $i = 7, 8, 9, 10$ ) versus  $1/R$  and find that the impact of the UED on the  $C_9$  is small. The suppression of  $|C_7|$  and  $|C_8|$ , that for  $1/R = 300$   $GeV$  amount to 82% and 66% relative to the SM values, respectively. For  $|C_{10}|$ , it can be enhanced by 16% for  $1/R = 300$   $GeV$ , which does not renormalize under QCD. The UED contribution to four quark QCD penguin operators are also neglected in this work because of rather smaller Wilson coefficients.

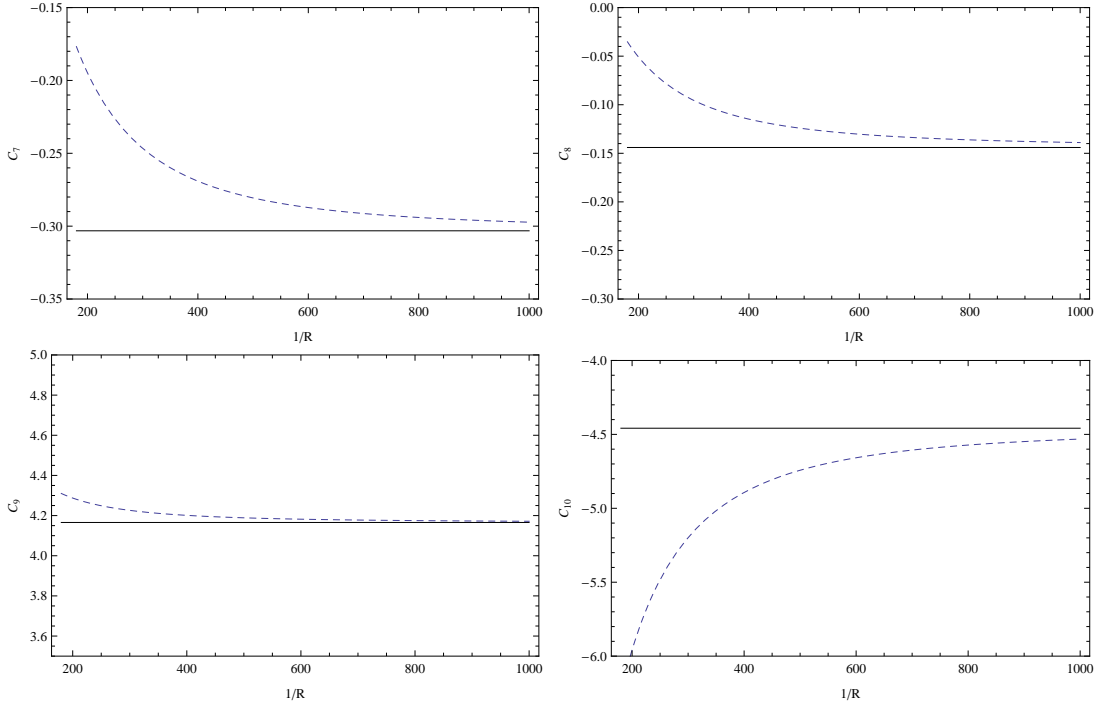


Figure 1: The wilson coefficients ( $\mu = 4.8 GeV$ ) plotted versus  $1/R$ . The constant lines are the SM values.

For the process  $B_s \rightarrow \phi \ell^+ \ell^-$ , the nonvanishing matrix elements are:

$$\begin{aligned} \langle \phi(k) | \bar{s} \gamma_\mu (1 - \gamma_5) b | \bar{B}_s(p) \rangle &= -i \varepsilon_\mu^* (m_{B_s} + m_\phi) A_1(q^2) + i(2p - q)_\mu (\varepsilon^* \cdot q) \frac{A_2(q^2)}{m_{B_s} + m_\phi} \\ &+ i q_\mu (\varepsilon^* \cdot q) \frac{2m_\phi}{q^2} [A_3(q^2) - A_0(q^2)] + \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p^\rho k^\sigma \frac{2V(q^2)}{m_{B_s} + m_\phi}, \end{aligned} \quad (6)$$

$$\begin{aligned}
\langle \phi(k) | \bar{s} \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | \bar{B}_s(p) \rangle &= i \varepsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu} p^\rho k^\sigma 2T_1(q^2) + T_2(q^2) \left[ \varepsilon_\mu^* (m_{B_s}^2 - m_\phi^2) - (\varepsilon^* \cdot q) (2p - q)_\mu \right] \\
&+ T_3(q^2) (\varepsilon^* \cdot q) \left[ q_\mu - \frac{q^2}{m_{B_s}^2 - m_\phi^2} (2p - q)_\mu \right], \tag{7}
\end{aligned}$$

where  $\varepsilon_\mu$  is the polarization vector of the  $\phi$  meson. By means of equation of motion, one can obtain several relations between form factors

$$A_3(q^2) = \frac{m_{B_s} + m_\phi}{2m_\phi} A_1(q^2) - \frac{m_{B_s} - m_\phi}{2m_\phi} A_2(q^2) \tag{8}$$

and  $A_0(0) = A_3(0), T_1(0) = T_2(0)$ . All signs are defined in such a way to render the form factors are real and positive. We will use the results calculated by using the technique of the light-cone QCD sum rule approach [8], the updated results and  $q^2$  dependence of the form factors can be found in Ref. [9]. In Ref.[7], the authors found that the uncertainties from the form factors can change slightly from their corresponding central values in the low  $s$  region, and highly suppressed in large  $s$  region, we will not consider the effect of these uncertainties here. The physical range in  $s = q^2$  extends from  $s_{\min} = 4m_l^2$  to  $s_{\max} = (m_{B_s} - m_\phi)^2$ .

Keeping the lepton mass and adopting the same convention and notation as [2], we find that the dilepton invariant mass spectrum for  $\bar{B}_s \rightarrow \phi \ell^+ \ell^-$  decay is given as

$$\frac{d\Gamma}{d\hat{s}} = \frac{G_F^2 \alpha_{em}^2 m_{B_s}^5}{2^{10} \pi^5} |V_{ts}^* V_{tb}|^2 \hat{u}(\hat{s}) D, \tag{9}$$

where the function  $D$  is defined as:

$$\begin{aligned}
D &= \frac{|A|^2}{3} \hat{s} \lambda (1 + 2 \frac{\hat{m}_\ell^2}{\hat{s}}) + |E|^2 \hat{s} \frac{\hat{u}(\hat{s})^2}{3} + \frac{1}{4\hat{m}_\phi^2} \left[ |B|^2 (\lambda - \frac{\hat{u}(\hat{s})^2}{3} + 8\hat{m}_\phi^2 (\hat{s} + 2\hat{m}_\ell^2)) + |F|^2 (\lambda - \frac{\hat{u}(\hat{s})^2}{3} + 8\hat{m}_\phi^2 (\hat{s} - 4\hat{m}_\ell^2)) \right] \\
&+ \frac{\lambda}{4\hat{m}_\phi^2} \left[ |C|^2 (\lambda - \frac{\hat{u}(\hat{s})^2}{3}) + |G|^2 \left( \lambda - \frac{\hat{u}(\hat{s})^2}{3} + 4\hat{m}_\ell^2 (2 + 2\hat{m}_\phi^2 - \hat{s}) \right) \right] - \frac{1}{2\hat{m}_\phi^2} \left[ \text{Re}(BC^*) (\lambda - \frac{\hat{u}(\hat{s})^2}{3}) (1 - \hat{m}_\phi^2 - \hat{s}) \right. \\
&\left. + \text{Re}(FG^*) ((\lambda - \frac{\hat{u}(\hat{s})^2}{3}) (1 - \hat{m}_\phi^2 - \hat{s}) + 4\hat{m}_\ell^2 \lambda) \right] - 2 \frac{\hat{m}_\ell^2}{\hat{m}_\phi^2} \lambda [\text{Re}(FH^*) - \text{Re}(GH^*) (1 - \hat{m}_\phi^2)] + \frac{\hat{m}_\ell^2}{\hat{m}_\phi^2} \hat{s} \lambda |H|^2. \tag{10}
\end{aligned}$$

With  $\hat{s} = s/m_B^2$ ,  $\hat{m}_\ell = m_l/m_B$  and  $\hat{m}_\phi = m_\phi/m_B$ , the kinematic variables are defined as

$$\lambda = 1 + \hat{m}_\phi^4 + \hat{s}^2 - 2\hat{s} - 2\hat{m}_\phi^2 (1 + \hat{s}) \tag{11}$$

$$\hat{u}(\hat{s}) = \sqrt{\lambda (1 - 4 \frac{\hat{m}_\ell^2}{\hat{s}})} \tag{12}$$

Combined the effective coefficients and form factors, the auxiliary functions  $A, B, C, E, F, G$  and  $H$  are referred to Ref. [2]. According to the definition of the forward-backward asymmetry (FBA), it is straightforward to obtain the expression of the normalized FBA as:

$$\frac{dA_{\text{FB}}^\phi}{d\hat{s}} D = \hat{u}(\hat{s}) \hat{s} [\text{Re}(BE^*) + \text{Re}(AF^*)] \tag{13}$$

We define the three orthogonal unit vectors in the center mass frame of dilepton as

$$\hat{e}_L = \vec{p}_+, \quad \hat{e}_N = \frac{\vec{p}_K \times \vec{p}_+}{|\vec{p}_K \times \vec{p}_+|}, \quad \hat{e}_T = \hat{e}_N \times \hat{e}_L, \tag{14}$$

which are related to the spin of lepton by a Lorentz boost. Then, the decay width of the  $B_s \rightarrow \phi \ell^+ \ell^-$  decay for any spin direction  $\hat{n}$  of the lepton, where  $\hat{n}$  is a unit vector in the dilepton center mass frame, can be written as:

$$\frac{d\Gamma(\hat{n})}{d\hat{s}} = \frac{1}{2} \left( \frac{d\Gamma}{d\hat{s}} \right)_0 [1 + (P_L \hat{e}_L + P_N \hat{e}_N + P_T \hat{e}_T) \cdot \hat{n}] \quad (15)$$

where the subscript "0" denotes the unpolarized decay width,  $P_L$  and  $P_T$  are the longitudinal and transverse polarization asymmetries in the decay plane respectively, and  $P_N$  is the normal polarization asymmetry in the direction perpendicular to the decay plane.

The lepton polarization asymmetry  $P_i$  can be obtained by calculating

$$P_i(\hat{s}) = \frac{d\Gamma(\hat{n} = \hat{e}_i)/d\hat{s} - d\Gamma(\hat{n} = -\hat{e}_i)/d\hat{s}}{d\Gamma(\hat{n} = \hat{e}_i)/d\hat{s} + d\Gamma(\hat{n} = -\hat{e}_i)/d\hat{s}}. \quad (16)$$

By a straightforward calculation, we get

$$P_{LD} = \sqrt{1 - 4 \frac{\hat{m}_l^2}{\hat{s}}} \left\{ \frac{2\hat{s}\lambda}{3} \text{Re}(AE^\dagger) + \frac{(\lambda + 12\hat{m}_\phi^2 \hat{s})}{3\hat{m}_\phi^2} \text{Re}(BF^\dagger) - \frac{\lambda(1 - \hat{m}_\phi^2 - \hat{s})}{3\hat{m}_\phi^2} \text{Re}(BG^\dagger + CF^\dagger) + \frac{\lambda^2}{3\hat{m}_\phi} \text{Re}(CG^\dagger) \right\}, \quad (17)$$

$$P_{ND} = \frac{-\pi\sqrt{\hat{s}}\hat{u}(\hat{s})}{4\hat{m}_\phi} \left\{ \frac{\hat{m}_l}{\hat{m}_\phi} [\text{Im}(FG^\dagger)(1 + 3\hat{m}_\phi^2 - \hat{s}) + \text{Im}(FH^\dagger)(1 - \hat{m}_\phi^2 - \hat{s}) - \text{Im}(GH^\dagger)\lambda] + 2\hat{m}_\phi \hat{m}_l [\text{Im}(BE^\dagger) + \text{Im}(AF^\dagger)] \right\}, \quad (18)$$

$$P_{TD} = \frac{\pi\sqrt{\lambda}\hat{m}_l}{4\sqrt{\hat{s}}} \left\{ 4\hat{s} \text{Re}(AB^\dagger) + \frac{(1 - \hat{m}_\phi^2 - \hat{s})}{\hat{m}_\phi^2} [-\text{Re}(BF^\dagger) + (1 - \hat{m}_\phi^2) \text{Re}(BG^\dagger) + \hat{s} \text{Re}(BH^\dagger)] + \frac{\lambda}{\hat{m}_\phi^2} [\text{Re}(CF^\dagger) - (1 - \hat{m}_\phi^2) \text{Re}(CG^\dagger) - \hat{s} \text{Re}(CH^\dagger)] \right\} \quad (19)$$

### 3 Numerical Analysis

In this section, we will examine the above mentioned physics observables and study their sensitivity to the compactification factor,  $1/R$ , which is the most important parameter in the single universal extra dimension model. Now, the parameters of the SM in our calculation are listed as follows:

$$\begin{aligned} m_{B_s} &= 5.36 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}, \quad m_c = 1.4 \text{ GeV}, \quad m_\phi = 1.02 \text{ GeV}, \quad m_\mu = 0.1057 \text{ GeV}, \\ m_\tau &= 1.7769 \text{ GeV}, \quad m_t = 172.4 \text{ GeV}, \quad m_W = 80.4 \text{ GeV}, \quad m_Z = 91.18 \text{ GeV}, \quad \sin^2 \theta_W = 0.23, \\ \alpha_{em} &= \frac{1}{137}, \quad \alpha_s(m_Z) = 0.118, \quad |V_{ts}^* V_{tb}| = 38.5 \times 10^{-3}, \quad \tau_{B_s} = 1.46 \times 10^{-12} \text{ s}. \end{aligned} \quad (20)$$

It is interesting to calculate the branching ratio of  $B_s \rightarrow \phi \gamma$  in the UED model firstly, since this channel will be measured easily among the  $B_s$  rare decays. Within the effective Hamiltonian, see equation (2), the decay rate of this channel is read:

$$\Gamma(B_s \rightarrow \phi \gamma) = \frac{\alpha G_F^2}{32\pi^4} |V_{tb} V_{ts}^*|^2 m_b^2 m_{B_s}^3 \left( 1 - \frac{m_\phi^2}{m_{B_s}^2} \right)^3 |C_7^{eff}|^2 |T_1(0)|^2 \quad (21)$$

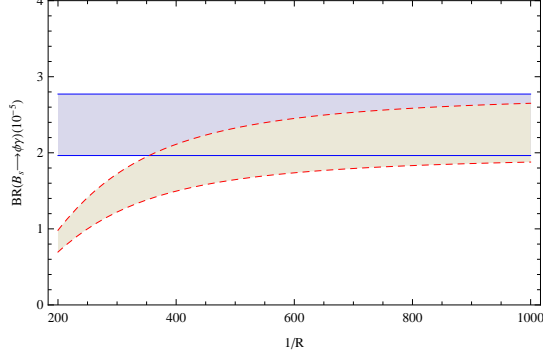


Figure 2: The branching ratio of  $B_s \rightarrow \phi\gamma$  changes with  $1/R$ , and horizontal band corresponds to the SM prediction.

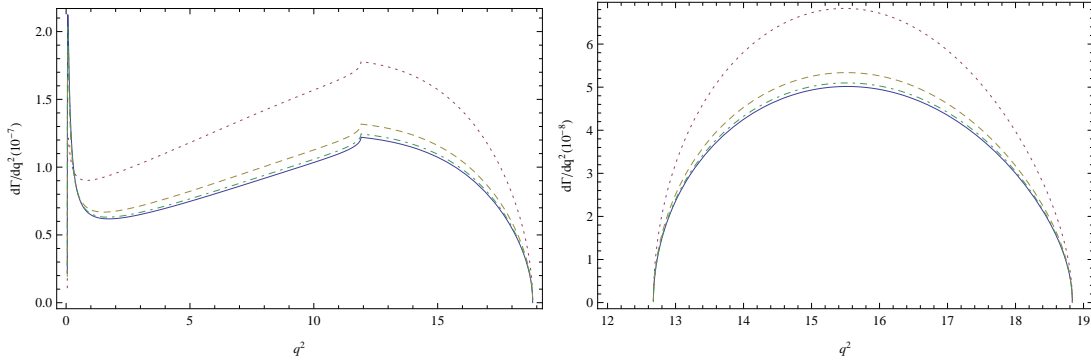


Figure 3: The variation of the dilepton invariant mass spectra of  $B_s \rightarrow \phi\mu^+\mu^-$  (left panel) and  $B_s \rightarrow \phi\tau^+\tau^-$  (right panel) with  $q^2$  (in units of  $\text{GeV}^2$ ). The solid line corresponds to the SM, dotted line, dashed line and dot-dashed line are for  $1/R = 200 \text{ GeV}, 500 \text{ GeV}, 1000 \text{ GeV}$  respectively.

In the SM, we found that the  $\text{BR}(B_s \rightarrow \phi\gamma) = (2.4 \pm 0.4) \times 10^{-5}$ , the major uncertainty is from the error of the form factor  $T_1(0)$ . Considering the contribution of the UED, we present the branching ratio dependence on compactification parameter  $1/R$  in Fig. 2. There are considerable discrepancies between the predictions of the UED and SM models for low values of the  $1/R$ . For  $1/R = 300 \text{ GeV}$ , the ratio can reach to  $1.5^{+0.2}_{-0.3} \times 10^{-5}$ . Such a discrepancy at low values of  $1/R$  can be a signal for the existence of extra dimensions. Accordingly, if the experiment can measure this channel well, we can constraint the range of  $1/R$ .

In term of Eq. (10), we illustrate the dilepton invariant mass spectra in Fig. 3. By integrating the differential ratios over  $q^2$ , in the SM, we obtain that:

$$\begin{aligned} \text{Br}(B_s \rightarrow \phi\mu^+\mu^-) &= 1.8 \times 10^{-6}; \\ \text{Br}(B_s \rightarrow \phi\tau^+\tau^-) &= 2.4 \times 10^{-7}. \end{aligned} \quad (22)$$

Considering the uncertainties in both theoretical side and experimental sides, the prediction of  $B_s \rightarrow \phi\mu^+\mu^-$  is consistent with CDF measurement  $\text{Br}(B_s \rightarrow \phi\mu^+\mu^-) = (1.44 \pm 0.56) \times 10^{-6}$  well, which is shown in Fig. 4. Note that

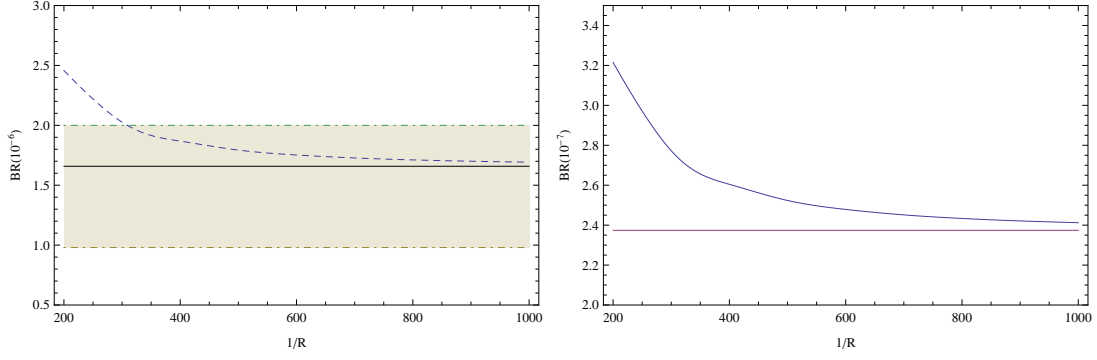


Figure 4: The branching ratio of  $B_s \rightarrow \phi \mu^+ \mu^-$  (left panel) and  $B_s \rightarrow \phi \tau^+ \tau^-$  (right panel) changes with  $1/R$ , and horizontal solid lines corresponds to the SM prediction. In the left panel, the band area depicts the experimental data from CDF.

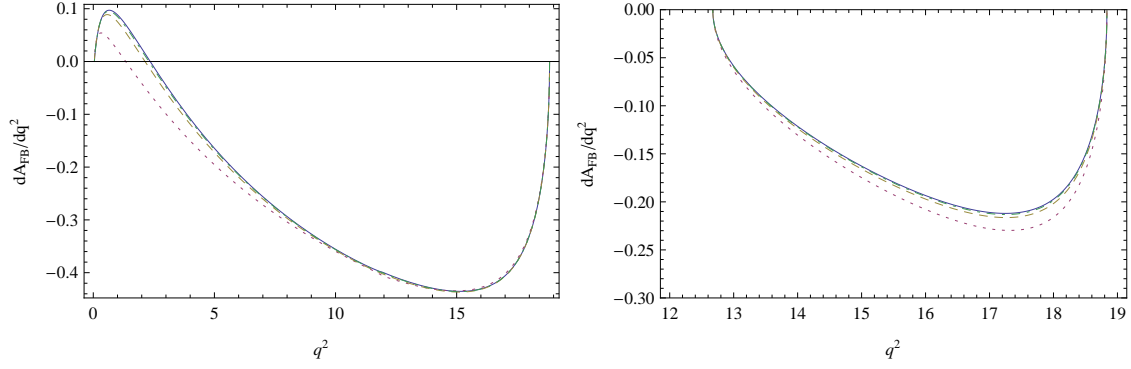


Figure 5: The lepton forward-backward asymmetry of  $B_s \rightarrow \phi \mu^+ \mu^-$  (left panel) and  $B_s \rightarrow \phi \tau^+ \tau^-$  (left panel) with  $q^2$  (in units of  $\text{GeV}^2$ ). The solid line corresponds to the SM, dotted line, dashed line and dot-dashed line are for  $1/R = 200 \text{ GeV}, 500 \text{ GeV}, 1000 \text{ GeV}$  respectively.

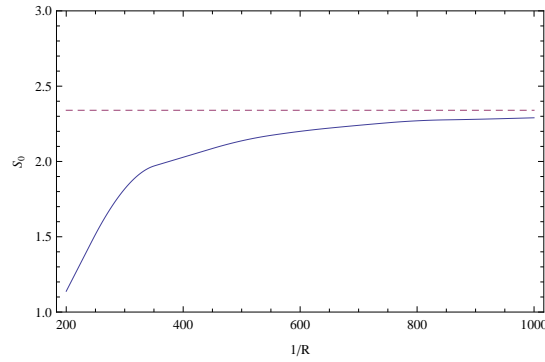


Figure 6: The zero of the FBA changes with  $1/R$ , and horizontal line corresponds to the SM prediction.



the small branching ratio of tau mode is due to highly suppressed phase space. We find there is difference between our results and these from the Ref. [7], that is because we ignore the resonance contribution and slight different parameter space. Adding the the UED contribution, from the figures, one can see that the extra dimension effects can increase both the differential width and branching ratios for low values of  $1/R$ . As  $1/R$  increases, this difference tends to diminish so that for higher values of  $1/R$  ( $1/R > 1000\text{GeV}$ ), the predictions of UED become very close to the results of SM. Such discrepancy at low values of  $1/R$  can be considered as a signal for the existence of extra dimensions. Moreover, once  $1/R > 500\text{GeV}$ , it is impossible to disentangle the extra dimension contribution clearly in both modes. As it is expected, the order of magnitudes of the branching ratios show a possibility to study such channels at the LHC.

We show the lepton forward-backward asymmetries (FBA) for both channels in Fig. 5. It is shown that there is also considerable discrepancy between the predictions of the UED and SM models for low values of  $1/R$ . As  $1/R$  increases, this difference starts to diminish. It should be stressed that the hadronic uncertainties almost have no influence on this symmetry, so it can provide the best tool to prob the new physics contribution. For the muon mode, due to the destructive interference between the photo penguin and Z penguin, the FBA should have a zero point  $s_0$ . Any deviation of the zero position  $s_0$  from that of the SM would give us the clue of the new physics. In Fig. 5, one can find that  $s_0$  decreases from the SM value. To make the situation clearer, the dependence of  $s_0$  on the compactification parameter  $1/R$  is depicted in Fig. 6, which shows that the zero position is pushed to a smaller value by decreasing  $1/R$ . Such a sensitivity indicates  $s_0$  is particularly suited to constrain the  $R$ . Hence, with the enhancement of experimental precision and statistics in LHC and CDF, the measurements of FBA would provide more data and effectively pine the NP effects.

Now, we turn to discuss the lepton polarization. In Fig. 7 and 8, we present the longitudinal and transverse polarization for  $B_s \rightarrow \phi \mu^+ \mu^-$  and  $B_s \rightarrow \phi \tau^+ \tau^-$ , respectively. For the normal polarization part, due to real  $C_{10}$ , it is the order of  $10^{-3}$  and cannot be observed even in the designed super-B factory, then we will ignore this part in this work. From the Fig. 7 of  $B_s \rightarrow \phi \mu^+ \mu^-$ , for small  $1/R$ , the deviation between UED model and SM is apparent. At the small value of momentum transfer, the  $|P_L|$  is larger than the SM prediction; while at  $q^2 > 3.2\text{GeV}^2$ , it will become smaller than that of SM. Thus, measurement of  $P_L$  will discriminate between the different models. For the  $P_T$  part, the predictions from different models are almost the same. As  $B_s \rightarrow \phi \tau^+ \tau^-$  as concerned, with smaller  $1/R = 200\text{GeV}$ , we can observe the deviation appears when  $q^2 > 16\text{GeV}^2$ . Again in the  $P_T$  part the predictions are the same as SM for all  $1/R$ .

## 4 Summary

Using form factors calculated within the light-cone sum rules, we presented the decay branching ratios of  $B_s \rightarrow \phi \gamma$  and  $B_s \rightarrow \phi \ell^+ \ell^-$  in a single universal extra dimension model (UED), which is view as one of the alternative theories beyond the standard model (SM) with only one new parameter. For the decay  $B_s \rightarrow \phi \ell^+ \ell^-$ , the dilepton invariant mass spectra, the forward-backward asymmetries, and double lepton polarizations are also calculated. For each case, we compared the obtained results with predictions of the SM. In lower values of the compactification factor  $1/R$ , we can

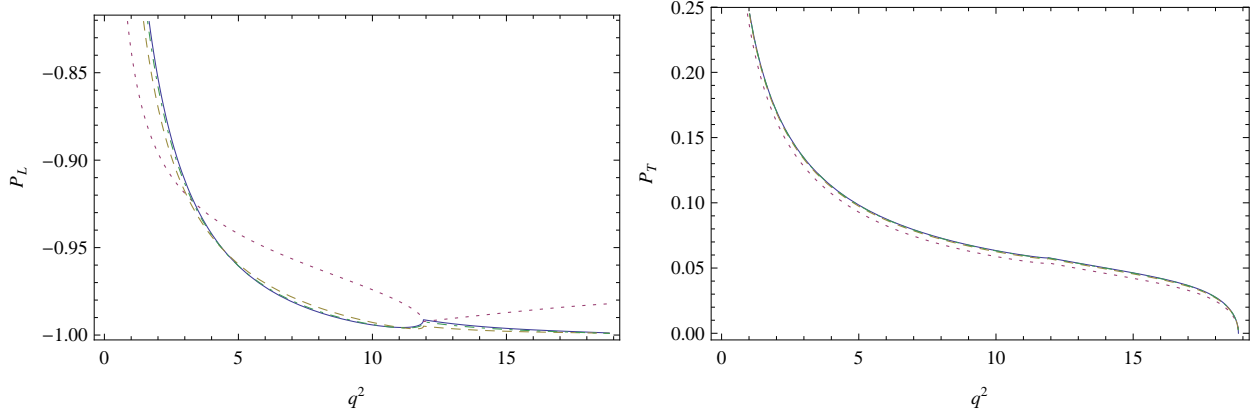


Figure 7: The dependence of longitudinal (left panel) and transverse(right panel) polarization of  $B_s \rightarrow \phi \mu^+ \mu^-$  with  $q^2$  (in units of  $\text{GeV}^2$ ). The solid line corresponds to the SM, dotted line, dashed line and dot-dashed line are for  $1/R = 200 \text{ GeV}, 500 \text{ GeV}, 1000 \text{ GeV}$  respectively.

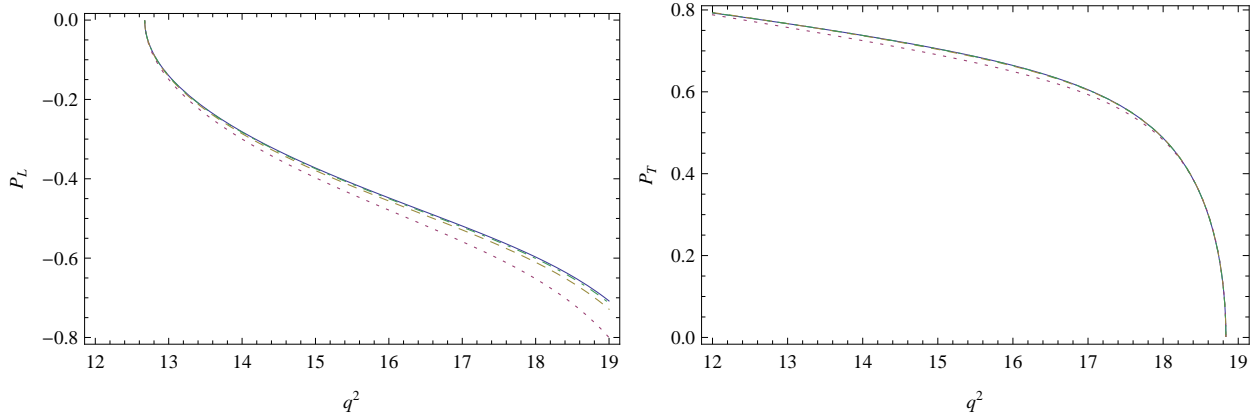


Figure 8: The dependence of longitudinal (left panel) and transverse(right panel) polarization of  $B_s \rightarrow \tau \mu^+ \mu^-$  with  $q^2$  (in units of  $\text{GeV}^2$ ). The solid line corresponds to the SM, dotted line, dashed line and dot-dashed line are for  $1/R = 200 \text{ GeV}, 500 \text{ GeV}, 1000 \text{ GeV}$  respectively.

see the considerable discrepancy between the UED and SM models. However, when  $1/R$  increases, the results of UED tend to diminish and at  $1/R = 1000$  GeV, two models have approximately the same predictions. For  $B_s \rightarrow \phi \mu^+ \mu^-$ , compared with data from CDF, the  $1/R$  tends to be larger than 350 GeV. We also note that the zero point of the forward-backward asymmetry become smaller, which will be an important plat to prob the contribution from the extra dimension. Moreover, the order of magnitude for branching ratios shows a possibility to study these channels at the Large Hadron Collider (LHC), CDF and the future super-B factory.

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